Mathematics I CBCGS _H(ESE) max marks :50

1. If x^2 & x^3 *in* $[1,2]$ then c of CMVT is [2]

$(a)\ \frac{14}{8}\quad (b)\ \frac{14}{9}\quad (c)\ \frac{14}{5}\quad (d)\ \frac{14}{3}$

2. The McLaurin Series expansion of $\tanh x$

$(a)x-\frac{x^3}{3}+\frac{2x^5}{15}-..\ \ (b)x+\frac{x^3}{3}+\frac{2x^5}{15}-..\ \ (c)x-\frac{x^3}{3!}+\frac{2x^5}{15!}-..\ \ (d)x^2-\frac{x^3}{3}+\frac{2x^5}{15}-..$ [1]

3. Expand $f(x)=x^2+1$ in the powers of (x-3)

$(a)\ 15+(x+3)+(x+3)^2\ \ (b)10+(x-3)6+(x-3)^2..$
$(c)10-(x-3)6-(x-3)^2...\ \ (d)\ 12-(x-3)6-(x-3)^2...$ [2]

4. *Evaluate* $\lim_{x\to 0}\ x^3\log x.$ [1]

$(a)\ \ 1\quad (b)\ \ -1\quad (c)\ \ 0\quad (d)\ \ 2$

5. Evaluate $\lim_{x\to 0}\ \left(\frac{1}{x}\right)^{2\sin x}.$ [2]

$(a)\ \ e\quad (b)\ \ e^{-1}\quad (c)\ \ 0\quad (d)\ \ 1$

6. If $z=x^2\ \tan^{-1}\left(\frac{y}{x}\right)$ then $\frac{\partial z}{\partial y}$ [2]

$(a)\ \ \frac{\partial z}{\partial y}=\frac{x^3}{x^2+y^2}-2y\ \tan^{-1}\left(\frac{x}{y}\right)\quad (b)\ \ \frac{\partial z}{\partial y}=\frac{y^3}{x^2+y^2}-2y\ \tan^{-1}\left(\frac{x}{y}\right)$

$(c)\ \ \frac{\partial z}{\partial y}=\frac{x^3}{x^2+y^2}-2y^2\ \tan^{-1}\left(\frac{x}{y}\right)\quad (d)\ \ \frac{\partial z}{\partial y}=\frac{1}{x^2+y^2}-2y^2\ \tan^{-1}\left(\frac{x}{y}\right)$

7. *If* $u=\log\left(\frac{x}{y}\right)$ *then find* u_x+u_y [1]

$(a)\frac{y+x}{xy}\ (b)\frac{y-x}{x^2y}\ (c)\frac{y-x}{xy^3}\ (d)\frac{y-x}{xy}$

8. *If* $u=f(r)$ *where* $r=t^2$ *then* $\frac{\partial u}{\partial t}$ [1]

$(a)\ t\,f'(r)\ (b)2t^2\,f'(r^2)\ (c)2t\,f'(r)\ \ (d)3t^2\,f'(r)$

9. If $\bar{r} = x\hat{i} + y\,\hat{j} + z\,\hat{k}$ *then Value of* $\nabla \bullet \bar{r}$ =?. [1]

$(a)\ 0\ (b)\,1\ (c)\,2\ \ (d)\,3$

10. In what direction from the point (2,1, -1) is the directional derivative of $\phi = x^2yz^3$ a maximum

$(a) 4\hat{i} + 4\hat{j} + 12\hat{k}\ \ (b) - 4\hat{i} - 4\hat{j} + 12\hat{k}\ \ (c) - 4\hat{i} - 3\hat{j} + 12\hat{k}\ \ (d) - 4\hat{i} - 4\hat{j} + 5\hat{k}$ [2]

11. If α is the 5th root of unity then $\alpha^5 = ?$ [1]

$(a)\ 0\ (b)\,1\ (c)\,2\ \ (d)\,3$

12. If $\cos\left(\theta + i\phi\right) = \cos\alpha + i\sin\alpha$ then $\cos\theta\cosh\phi = ?$ [2]

$(a)\ \cos\alpha\ \ (b)\,\sin\alpha\ \ (c)\,2\ \ (d)\,3$

13. *Solve the equation for real values of* $x,\ 17\cosh x + 18\sinh x = 1.$ [2]

$(a)\,\log 5\ \ (b) - \log 5\ \ (c) - \log 4\ \ (d) - \log 3$

14. The Logarithmic Value of $\tanh^{-1} x$ [1]

$(a)\,\frac{1}{3}\log\left(\frac{1+x}{1-x}\right)\ (b)\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)\ \ (c)\frac{1}{2}\log\left(\frac{1-x}{1+x}\right)\ \ (d)\log\left(\frac{1+x}{1-x}\right)$

15. On Separating in to real and Imaginary part the value of $(2i)^{2i}$ is [2]

$(a) -\pi + 4i\ \ (b) -\pi + \log 4i\ \ \ (c) -\pi + \log 2i\ \ \ (d) -\pi + i$

16. With the root test, test the convergence of $\frac{1}{n^n}$ [2]

(a) *convergent* (b) *Divergent* (c) *Oscilatory* (d) *Test fails*

17. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then rank of A is [2]

$(a)\ 0\ \ (b) 2\ \ (c) 3\ \ \ (d) 1$

18. If $A = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$ then A^{θ} is [2]

$(a)\begin{bmatrix} -3i & 1-i & -3+2i \\ -1-i & i & -1-2i \\ 3+2i & 1-2i & 0 \end{bmatrix}$ $(b)\begin{bmatrix} -3i & 1-i & -3+2i \\ -1-i & -i & -1-2i \\ 3+2i & 1-2i & 0 \end{bmatrix}$

$(c)\begin{bmatrix} -3i & 1+i & -3+2i \\ -1-i & -i & -1-2i \\ 3+2i & 1-2i & 0 \end{bmatrix}$ $(d)\begin{bmatrix} -3i & 1-i & -3+2i \\ -1-i & i & -1-2i \\ 3+2i & 1+2i & 0 \end{bmatrix}$

19. A Homogeneous System of Equation is always consistent as its one solution is always? [1]

$(a)(2,2,2...)(b)(0,0,0...)\ (c)(-1,-1,-1...)\ (d)(1,1,1...)$

20. A matrix has dimension 9×7 then its maximum rank possible is [1]

$(a)9(b)7\ (c)5\ (d)3$

21. Evaluate $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\ dx$ [2]

$(a)\ \frac{1}{2}\left|\frac{1}{4}\right.\ (b)\frac{1}{2}\left|\frac{3}{4}\right.\ (c)\frac{1}{2}\left|\frac{5}{4}\right.\ \ (d)\frac{1}{2}\left|\frac{1}{2}\right.$

22. The Value of $B(3,2)$ [2]

$(a)\ \frac{1}{12}\ (b)\frac{5}{12}(c)\frac{1}{2}\ \ (d)\frac{3}{2}$

23.The value of $\left|-\frac{3}{2}\right.$ is [2]

$(a)\ \frac{4}{3}\sqrt{\pi}\ (b)-\frac{4}{3}\sqrt{\pi}\ (c)\frac{2}{3}\sqrt{\pi}\ \ (d)\frac{1}{3}\sqrt{\pi}$

24. Evaluate $\int_0^\infty 3^{-4x^2}\,dx$ [2]

(a) $\frac{\sqrt{\pi}}{2\sqrt{\log 3}}$ (b) $\frac{\sqrt{\pi}}{4\sqrt{\log 3}}$ (c) $\frac{\sqrt{\pi}}{3\sqrt{\log 3}}$ (d) $\frac{\sqrt{\pi}}{\sqrt{\log 3}}$

25. Evaluate $\int_0^\infty \frac{x^2}{(1+x)^7}\,dx$ [2]

(a) $\frac{1}{60}$ (b) $\frac{1}{30}$ (c) $\frac{1}{40}$ (d) $\frac{1}{120}$

26. If $B(n,2)=\frac{1}{12}$ and n is a + ve integer, find n. [2]

$(a)\ 3 \qquad (b)\ 5 \qquad (c)\ 1 \qquad (d)\ 2$

27. If f(z)=sin z then its real part u(x,y) is [2]

$(a)\ \sin x\cosh y \qquad (b)\ \cos x\cosh y \qquad (c)\ \cos x\sinh y \qquad (d)\ \sin x\sinh y$

28. If f(z)=logz then its imaginary part v(x,y) is [2]

$(a)\ \log xy \qquad (b)\ 1/2\log(x^2+y^2) \qquad (c)\ \tan^{-1}(\frac{y}{x}) \qquad (d)\ 0$

29. If Real part of an analytic function is $u(x,y)=e^x\cos y$ then its imaginary part is [2]

$(a)\ e^y\cos x \qquad (b)\ e^x\cos x \qquad (c)\ e^x\sin y \qquad (d)\ e^x\sin x$

30. If $f(z)=\sin z$ is analytic then $f'(z)$ is [1]

$(a)\ \tan z \quad (b)\cos z \quad (c)\ Not\ \ Defined \quad (d)\,0$